WILSON COEFFICIENT FOR VACUUM CONDENSATE OF PHOTONS FROM ELECTRODYNAMICS PHOTON PROPAGATOR

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Abstract

In the framework of the operator product expansion I provide a detailed evaluation of the contribution of the lowest photon condensate to the two-point photon Green function for four-dimensional spinor electrodynamics and three-dimensional scalar electrodynamics. Since the above mentioned condensate affects the part of the propagator contributing to physical amplitudes I suggest that this condensate could play a role in the non-perturbative dynamics of Abelian gauge field theories.

1 Introduction

Matrix elements of composite operators can bring non-perturbative information about the dynamics of some quantum field model. It was pointed out in [1, 2] that superrenormalizable theories cure their infrared divergences if one allows for coupling constant logarithms in the perturbative expansion with remaining non-perturbative terms determined by matrix elements of composite operators. This analysis was performed for three dimensional electrodynamics, either scalar or spinor, and for both of the cases it was concluded that the vacuum expectation value of the square of the photon field, $< A^2 >$, provides a non-perturbative contribution to the self-energy of matter fields. It was moreover argued that due to the lack of gauge invariance this expectation value cannot contribute to photon propagator. Since recently it has been proved the gauge invariance of $< A^2 >$ in gauge field theories either Abelian or non-Abelian [3], I will check if the above mentioned condensate can affect physical gauge invariant amplitude in which photon propagator is sandwiched between conserved currents. In this letter I will calculate the Wilson coefficient for $< A^2 >$ from the operator product expansion of photon propagator in four-dimensional scalar electrodynamics and three dimensional scalar electrodynamics [4]. I will use the plane wave method [5], shown to be powerful for QCD-sum rules but totally equivalent to other methods present on the market, as for example the Covariant gauge coordinates space method. [6]

2 Wilson coefficient for $< A^2 >$ in four-dimensional spinor electrodynamics

I consider massive fermions ψ interacting with a massless Abelian gauge field A_μ in four dimensions Minkowskyan space-time ,

$$S_{\mathrm{QED}} = \int d^4x \left[\imath\bar{\psi}(\widehat{\partial_\mu} - \imath e\widehat{A_\mu} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right] \quad (1)$$

$$\begin{aligned}(\widehat{\partial_\mu} - \imath e\widehat{A_\mu}) &= \gamma^\mu(\partial_\mu - \imath e A_\mu) \\ F^{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu\end{aligned}$$

The coupling e is dimensionless; the interaction is renormalizable.

The aim of this section is to compute the coefficient of the vacuum condensate $< 0|A^2|0 >$ in the operator product expansion, as $x \to 0$, of

$$< 0|TA_\mu(x)A_\nu(0)|0 >\sim \sum_O C_O(x^2) < 0|[O(0)]|0 > . \tag{2}$$

where $[O(0)]$ denotes some renormalized operator.

First I consider the tree graphs for

$$< 0|TA_\mu(x)A_\nu(0)A_\tau(p_1)A_\sigma(p_2)|0 > . \tag{3}$$

These give

$$\frac{\imath^2}{p_1^2 p_2^2}(g_{\mu\tau}g_{\nu\sigma}\exp(\imath p_1 x) + g_{\mu\sigma}g_{\nu\tau}\exp(\imath p_2 x)) \tag{4}$$

and since I am interested in the insertion of $< A^2 >$ in the photon propagator, I will consider the previous amplitude after the contraction with $g_{\mu\nu}$

$$\frac{\imath^2}{p_1^2 p_2^2}g_{\sigma\tau}(\exp(\imath p_1 x) + \exp(\imath p_2 x)). \tag{5}$$

Expansion in a power series about $x = 0$ gives

$$\frac{\imath^2}{p_1^2 p_2^2}g_{\sigma\tau}(2 + \imath(p_1 + p_2)x + ...), \tag{6}$$

this is equivalent to replace

$$TA_\mu(x)A_\nu(0) = \frac{g_{\mu\nu}}{4}A^2(0) + \frac{g_{\mu\nu}}{8}x^\rho\partial_\rho A^2...... \tag{7}$$

Since (7) is an operator equality, it is valid for all its matrix elements, in particular the vacuum expectation value $< A^2 >$ affects photon propagator.

The first non trivial contribution to the two-point function of $TA_\mu(x)A_\nu(0)$ is e^4-order and it is given by the following sum of six Feynman diagrams:

$$\begin{aligned}&-\frac{e^4}{24p_1^2p_2^2}\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{\exp(-\imath qx)}{q^2(q+p_1+p_2)^2} \\ &\Big\{\mathrm{Tr}\left[\gamma_\nu\frac{1}{\widehat{k}-\widehat{q}-m}\gamma_\mu\frac{1}{\widehat{k}-m}\gamma_\sigma\frac{1}{\widehat{k}+\widehat{p_2}-m}\gamma_\tau\frac{1}{\widehat{k}+\widehat{p_1}+\widehat{p_2}-m}\right] \\ &+\mathrm{Tr}\left[\gamma_\nu\frac{1}{\widehat{k}+\widehat{p_1}+\widehat{p_2}+\widehat{q}-m}\gamma_\mu\frac{1}{\widehat{k}+\widehat{p_1}+\widehat{p_2}-m}\gamma_\tau\frac{1}{\widehat{k}+\widehat{p_2}-m}\gamma_\sigma\frac{1}{\widehat{k}-m}\right] \\ &+\mathrm{Tr}\left[\gamma_\tau\frac{1}{\widehat{k}-\widehat{q}-m}\gamma_\mu\frac{1}{\widehat{k}-m}\gamma_\sigma\frac{1}{\widehat{k}+\widehat{p_2}-m}\gamma_\nu\frac{1}{\widehat{k}-\widehat{q}-\widehat{p_1}-m}\right] \\ &+\mathrm{Tr}\left[\gamma_\nu\frac{1}{\widehat{k}-\widehat{p_2}-\widehat{q}-m}\gamma_\sigma\frac{1}{\widehat{k}-\widehat{q}-m}\gamma_\mu\frac{1}{\widehat{k}-m}\gamma_\tau\frac{1}{\widehat{k}+\widehat{p_1}-m}\right] \\ &+\mathrm{Tr}\left[\gamma_\nu\frac{1}{\widehat{k}-\widehat{p_1}-\widehat{p_2}-\widehat{q}-m}\gamma_\sigma\frac{1}{\widehat{k}-\widehat{q}-\widehat{p_1}-m}\gamma_\tau\frac{1}{\widehat{k}-\widehat{q}-m}\gamma_\mu\frac{1}{\widehat{k}-m}\right] \\ &+\mathrm{Tr}\left[\gamma_\nu\frac{1}{\widehat{k}-\widehat{p_1}-\widehat{p_2}-\widehat{q}-m}\gamma_\tau\frac{1}{\widehat{k}-\widehat{q}-\widehat{p_2}-m}\gamma_\sigma\frac{1}{\widehat{k}-\widehat{q}-m}\gamma_\mu\frac{1}{\widehat{k}-m}\right]\Big\}.\end{aligned} \tag{8}$$

The expression used here and in the following for the external legs of the photon propagator,

$$\frac{\imath g_{\mu\nu}}{q^2}, \tag{9}$$

doesn't take into account gauge-dependent terms, because they don't contribute to $S-$matrix elements when photon-propagator is sandwiched between two conserved currents.

When $x \to 0$ the integral on q in (8) diverges as a logarithm, since, as it will be proved, the integral on k is ultraviolet and infrared finite. This logarithmic divergence is a symptom of the fact that there are two important region of q that contribute. The first is where q is finite as $x \to 0$, the graphs receive contribution by replacing $TA_\mu(x)A_\nu(0)$ by $\frac{g_{\mu\nu}}{4}A^2(0)$. The second region is where q becomes large, up to $O(1/x)$ as $x \to 0$; in this region the interaction vertex in coordinate space is close to x and 0. In the second region the loop is confined to a small region in coordinate space. From the point of view of p_1 and p_2 the loop is a point, therefore we should be able to represent the contribution of this region by an extra term in the Wilson coefficient of A^2:

$$TA_\mu(x)A_\nu(0) \sim C_{A^2}(x)\frac{g_{\mu\nu}}{4}[A^2(0)] + ... \qquad (10)$$

$$C_{A^2}(x) = 1 + e^4 c(x^2) \qquad (11)$$

I shall calculate $c(x^2)$. Now the contribution of the first region is given by replacing $TA_\mu(x)A_\nu(0)$ by $\frac{g_{\mu\nu}}{4}[A^2(0)]$. So let me add and subtract the renormalized Green function of the renormalized operator $[A^2(0)]$:

$$\frac{g_{\mu\nu}}{4} < 0|T[A^2(0)]A_\tau(p_1)A_\sigma(p_2)|0 > . \qquad (12)$$

The Green function provide the contribution to the Wilson coefficient of order 1 from the first region. The reminder contribution of order e^4 is:

$$\begin{aligned} &-\frac{e^4}{48p_1^2p_2^2}\int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{\exp(-\imath qx)-1}{(q^2)^2(k-q)^2(k^2)^3}\times \\ &\mathrm{Tr}\left[\gamma^\mu(\widehat{k}-\widehat{q})\gamma_\mu\widehat{k}\gamma_\sigma\widehat{k}\gamma_\tau\widehat{k}+\gamma^\mu(\widehat{k}-\widehat{q})\gamma_\mu\widehat{k}\gamma_\tau\widehat{k}\gamma_\sigma\widehat{k}\right]+ \\ &-\frac{e^4}{96p_1^2p_2^2}\int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{\exp(-\imath qx)-1}{(q^2)^2[(k-q)^2]^2(k^2)^2}\times \\ &\mathrm{Tr}\left[\gamma^\mu(\widehat{k}-\widehat{q})\gamma_\tau(\widehat{k}-\widehat{q})\gamma_\mu\widehat{k}\gamma_\sigma\widehat{k}+\gamma^\mu(\widehat{k}-\widehat{q})\gamma_\sigma(\widehat{k}-\widehat{q})\gamma_\mu\widehat{k}\gamma_\tau\widehat{k}\right] \end{aligned} \qquad (13)$$

where it has been put $p_1 = p_2 = m = 0$, for the reason explained in the following.

Since to the lowest order

$$< 0|TA^2A_\tau(p_1)A_\sigma(p_2)|0 >= \frac{-2g_{\sigma\tau}}{p_1^2p_2^2}, \qquad (14)$$

it is possible to identify $c(x^2)$ as the $x \to 0$ behaviour of (13). That leading power behaviour is independent on p_1, p_2, m, as it can be easily seen by differentiating (13) with p_1, p_2, $m \neq 0$, with respect to any of these variables. The result is a convergent integral which goes to zero like a power of x when $x \to 0$. The definition of $c(x^2)$ is therefore at $p_1 = p_2 = m = 0$.

After making some $\gamma-$gymnastic (13) becomes:

$$\begin{aligned} &-\frac{e^4}{24p_1^2p_2^2}\int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{\exp(-\imath qx)-1}{(q^2)^2(k-q)^2(k^2)^3}\times \\ &[8g_{\sigma\tau}k^4 - 16k^2k_\sigma k_\tau - 8g_{\sigma\tau}k^2(kq) + 32(kq)k_\sigma k_\tau - 8k^2k_\tau q_\sigma - 8k^2k_\sigma q_\tau] \end{aligned} \qquad (15)$$

$$\begin{aligned} &-\frac{e^4}{48p_1^2p_2^2}\int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{\exp(-\imath qx)-1}{(q^2)^2((k-q)^2)^2(k^2)^2}\times \\ &[16k^2(k-q)_\tau(k-q)\sigma - 16k_\sigma k_\tau(k-q)^2 - 8g_{\sigma\tau}k^2(k-q)^2]. \end{aligned} \qquad (16)$$

Let me fix on $k-$integral. A first contribution to the sum (15) and (16) is given by the sum of potential logarithmic divergent terms:

$$-\frac{e^4}{24p_1^2p_2^2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{8g_{\sigma\tau}k^4 - 16k^2k_\sigma k_\tau}{(k^2)^3(k-q)^2} - \frac{4g_{\sigma\tau}(k^2)^2}{(k^2)^2[(k-q)^2]^2}\right]. \qquad (17)$$

Integrals can be performed by introducing the Feynman-parameter x

$$-\frac{\imath e^4q^2}{384\pi^2p_1^2p_2^2}\left(-\frac{1}{2}\times(-8)-4\right)\lim_{\varepsilon\to 0}\Gamma(\varepsilon) \qquad (18)$$

$$-\frac{\imath e^4q^2}{48\pi^2p_1^2p_2^2}\int_0^1 dx(x-1)\left[\frac{x^2g_{\sigma\tau}}{(x^2q^2-xq^2)} - 2\frac{x^2q_\sigma q_\tau}{(x^2q^2-xq^2)}\right] \qquad (19)$$

and logarithmic divergences cancel each others [7], giving the finite result:

$$-\frac{\imath}{4\pi^2}\frac{e^4}{p_1^2p_2^2}\left(g_{\sigma\tau}-2\frac{q_\sigma q_\tau}{q^2}\right). \tag{20}$$

The following finite contribution remains in (15):

$$\begin{aligned}-\frac{e^4}{24p_1^2p_2^2}\quad & \int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2)^3(k-q)^2}\times \\ & \left[-8g_{\sigma\tau}k^2(kq)+32k_\sigma k_\tau(kq)-8k^2k_\tau q_\sigma-8k^2k_\sigma q_\tau\right]\end{aligned} \tag{21}$$

and turning these integrals into a Gaussian form, after using Feynman parameters one easily gets:

$$\frac{\imath e^4}{96\pi^2p_1^2p_2^2}g_{\sigma\tau}. \tag{22}$$

The remaining terms in (16) don't add any contributions because:

$$\int\frac{d^4k}{(2\pi)^4}\left[\frac{16k^2(k-q)_\sigma(k-q)_\tau}{(k^2)^2[(k-q)^2]^2}-\frac{16(k-q)^2k_\sigma k_\tau}{(k^2)^2[(k-q)^2]^2}\right]=0, \tag{23}$$

as it can be easily seen by making the change of variables $k\to k+q$ and observing that the two integrals are even functions of the four-vector q or by explicit integration using Feynman parameters. Moreover

$$\begin{aligned}&-\frac{e^4}{48p_1^2p_2^2}\int\frac{d^4k}{(2\pi)^4}\frac{-8g_{\sigma\tau}k^2q^2+16g_{\sigma\tau}k^2(kq)}{(k^2)^2[(k-q)^2]^2}= \\ &-\frac{g_{\sigma\tau}e^4}{48p_1^2p_2^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2}+\frac{g_{\sigma\tau}e^4}{48p_1^2p_2^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{[(k-q)^2]^2}= \\ &\frac{\imath}{16\pi^2}\lim_{\varepsilon\to0}\Gamma(\varepsilon)-\frac{\imath}{16\pi^2}\lim_{\varepsilon\to0}\Gamma(\varepsilon)=0,\end{aligned} \tag{24}$$

where last line has been obtained after the integration through Feynman-parameters. The sum of all contributions is:

$$\begin{aligned}&\frac{2\imath e^4}{192\pi^2p_1^2p_2^2}\int\frac{d^4q}{(2\pi)^4}\frac{\exp(-\imath qx)-1}{(q^2)^2}\frac{q_\sigma q_\tau}{q^2} \\ &=\frac{2\imath e^4g_{\sigma\tau}}{16\pi^2p_1^2p_2^2}\int\frac{d^4q}{(2\pi)^4}\frac{\exp(-\imath qx)-1}{(q^2)^2}+......,\end{aligned} \tag{25}$$

where dots indicate terms vanishing as $x\to0$.

The integral can be done by using

$$\frac{1}{(q^2)^2}=\int_0^{+\infty}dz\exp(-z(-q^2))z \tag{26}$$

The e^4-order Wilson coefficient for $<A^2>$ in four-dimensional spinor quantum electrodynamics is:

$$1+\frac{\imath e^4}{6144\pi^4}\left(\gamma+\log\left(-\frac{x^2}{4}\right)\right). \tag{27}$$

3 Wilson coefficient for $<A^2>$ in three-dimensional scalar electrodynamics

In this section I will confirm a non-zero result for the Wilson coefficient of $<A^2>$, also for three-dimensional scalar electrodynamics. Although the logic that I am going to follow is the same than in the previous section, here I will work in the momentum representation. Divergent integral will be regularized by analytical continuation as explained in the note at the end of this section.

The action is

$$S = \int d^3x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |(\partial_\mu + \imath e A_\mu)\varphi|^2 \right], \tag{28}$$

which describes the dynamics of massless scalar field φ interacting with photons A_μ. The square of the coupling constant e ha dimensions of mass; the interaction is super-renormalizable.

The Wilson coefficient for $< A^2 >$ is again obtained from the $q^2 \to \infty$ behaviour of the amplitude:

$$g^{\mu\nu} < 0|T A_\mu(q) A_\nu(0) A_\tau(p_1) A_\sigma(p_2)|0 > . \tag{29}$$

To lowest order one has

$$< 0|T A^2 A_\tau(p_1) A_\sigma(p_2)|0 >= \frac{-2g_{\sigma\tau}}{p_1^2 p_2^2}. \tag{30}$$

The higher corrections to the Green function (29) involve three kinds of diagrams.

The sum of diagrams of the first type gives:

$$\begin{aligned} &\frac{4e^4 g_{\tau\sigma}}{p_1^2 p_2^2} \frac{1}{(q^2)(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2(k-p_1-q)^2} + \\ &\frac{4e^4 g_{\tau\sigma}}{p_1^2 p_2^2} \frac{1}{(q^2)(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2(k-p_2-q)^2} + \\ &\frac{2e^4 g_{\tau\sigma}}{p_1^2 p_2^2} \frac{1}{(q^2)(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2(k+p_1+p_2)^2}. \end{aligned} \tag{31}$$

It contributes to Wilson coefficient by

$$-\frac{e^4}{2} \frac{1}{(q^2)^2 \sqrt{q^2}}. \tag{32}$$

All integrals are convergent.The last term in (31) contributes to the Wilson coefficient of the expectation value of a non-local operator determined by the action of $\frac{1}{\sqrt{(p_1+p_2)^2}}$ on $[A^2(0)]$.

The sum of diagrams of second type is:

$$\begin{aligned} &-\frac{2e^4 g_{\sigma\tau}}{p_1^2 p_2^2} \frac{1}{q^2(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{(2k-q)_\mu (2k-q-p_1-p_2)^\mu}{k^2(k-q-p_1-p_2)^2(k-q)^2} \\ &-\frac{2e^4}{p_1^2 p_2^2} \frac{1}{q^2(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{(2k+p_1)_\tau (2k-q)_\sigma}{k^2(k+p_1)^2(k-q)^2} \\ &-\frac{2e^4}{p_1^2 p_2^2} \frac{1}{q^2(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{(2k+p_2)_\sigma (2k-q)_\tau}{k^2(k+p_2)^2(k-q)^2} \\ &-\frac{2e^4}{p_1^2 p_2^2} \frac{1}{q^2(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{(2k+p_1)_\tau (2k+q+p_1+p_2)_\sigma}{k^2(k+p_1)^2(k+q+p_1+p_2)^2} \\ &-\frac{2e^4}{p_1^2 p_2^2} \frac{1}{q^2(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \frac{(2k+p_2)_\sigma (2k+q+p_1+p_2)_\tau}{k^2(k+p_2)^2(k+q+p_1+p_2)^2} + \ldots\ldots. \end{aligned}$$

dots mean contributions to (29), which doesn't play any role in the computation of this Wilson coefficient.

After regularization of (33) the contribution to the Wilson coefficient is equal to

$$\frac{7}{12} e^4 \frac{1}{(q^2)^2 \sqrt{q^2}}. \tag{33}$$

The third type of diagrams gives:

$$\begin{aligned} &\frac{e^4}{6p_1^2 p_2^2} \frac{1}{q^2(q+p_1+p_2)^2} \int \frac{d^3k}{(2\pi)^3} \\ &\left\{ \frac{(2k+p_1)_\tau (2k-p_2)_\sigma (2k+2p_1+q) \cdot (2k+q+p_1-p_2)}{k^2(k+p_1)^2(k+q+p_1)^2(k-p_2)^2} \right. \end{aligned}$$

$$+\frac{(2k+p_1)_\tau(2k+2q+2p_1+p_2)_\sigma(2k+2p_1+q)\cdot(2k+q+p_2+p_1)}{k^2(k+p_1)^2(k+q+p_1)^2(k+q+p_1+p_2)^2}$$
$$\left.+\frac{(2k+p_2)_\sigma(2k-p_1-2q)_\tau(2k-q)\cdot(2k-q-p_1+p_2)}{k^2(k+p_2)^2(k-p_1-q)^2(k-q)^2}\right\}$$
$$+\left\{p_1\leftrightarrow p_2\ \bigwedge\ \sigma\leftrightarrow\tau\right\}. \tag{34}$$

Its contribution to Wilson coefficient is

$$-\frac{241}{192}e^4\int\frac{d^3q}{(2\pi)^3}\frac{\exp(-\imath qx)-1}{(q^2)^2\sqrt{q^2}}. \tag{35}$$

Finally the Wilson coefficient in the momentum space for $< A^2 >$ for three-dimension scalar quantum electrodynamics is

$$-\frac{75}{64}e^4\frac{1}{(q^2)^2\sqrt{q^2}}. \tag{36}$$

3.1 Note about the regularization of some divergent integrals

The integration on k in the previous section can be done by using Feynman parameters. All integrand are integrable in the infrared and in the ultraviolet but they have non-integrable singularities for $k=q$. The renormalized values of these integrals can be obtained by analytical continuation [8], which is applied in the following way. First one introduces into the initial divergent integral a parameter in such a way that for some values of this parameter that integral becomes convergent. Certainly these finite values of modified integral have no (direct) relation to that one of initial divergent integral. However, if the modified integral is an analytic function of the regularization parameter, that integral is defined through this function in its analyticity domain. Further one postulates that the renormalized (or physical) value of the initial divergent integral is equal to the respective value of this new analytic function. Let us consider the regularization of the following integral as representative of the all others:

$$\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2[(k-q)^2]^2} \tag{37}$$

Due to the bad non-integrable singularity at $k=q$, I consider the following integral depending on the complex parameter h

$$\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2[(k-q)^2]^h} \tag{38}$$

By introducing Feynman parameters:

$$\frac{1}{k^2((k-q)^2)^h}=-\imath^{-2h}\frac{\Gamma(h+1)}{\Gamma(h)}\int_0^1\frac{x^{h-1}}{(-k^2+2xkq-xq^2)^{h+1}}dx \tag{39}$$

therefore (38) becomes

$$-\imath^{-2h}\frac{\Gamma(h+1)}{\Gamma(h)}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{x^{h-1}}{(-k^2+2xkq-xq^2)^{h+1}} \tag{40}$$

The integration on k gives:

$$-\imath^{-2h}\frac{1}{\Gamma(h)}\int_0^1 dx\frac{\Gamma\left(h-\frac{1}{2}\right)}{(4\pi)^{3/2}}\frac{x^{h-1}}{(x^2q^2-xq^2)^{h-\frac{1}{2}}} \tag{41}$$

and integrating on x

$$-\frac{\imath^{-4h+1}\pi}{(4\pi)^{3/2}(q^2)^{h-1/2}\sin\left[\pi\left(h-\frac{1}{2}\right)\right]\Gamma(h)}\frac{1}{\Gamma(h-2)} \tag{42}$$

for $Re[h]<\frac{3}{2}$. By avoiding poles at seminteger values of h, the value of this integral can be analytically continued in a neighbourhood of $h=2$, and I conclude that the regularized value of (38) is

$$\left[\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2((k-q)^2)^2}\right]_{\text{reg.}}=0. \tag{43}$$

4 Conclusions

In this letter I have proved that photon propagator of four-dimensional spinor electrodynamics and three-dimensional scalar electrodynamics might receive contributions from the condensate $< A^2 >$ because its Wilson coefficient is not zero. Previous works claimed the cancellation of the contribution of this condensate in photon propagator due to a - not at all there- gauge invariance of this Green function. This non-zero result is therefore totally expected also from the non-Abelian analysis provided by Lavelle et all.[9]. Here I have found that the contribution of $< A^2 >$ might affect also that part of the photon propagator which enters directly in the construction of gauge invariant physical amplitudes. In particular, by using the plane wave method to calculate physical amplitudes, one realizes that the condensate $< A^2 >$ contributes to the amplitude of four photons. It would be interesting to understand the possible role played by $< A^2 >$ in the phenomenology of the hard-scattering light by light.

The contribution of $< A^2 >$ to such a physical amplitude can be considered a first experiment to check the gauge invariance of this condensate. It will be the subject of next investigations to compute directly this condensate in gauge theories in a non trivial background, as for instance an instanton, in order to check its gauge invariance beyond perturbation theory.

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